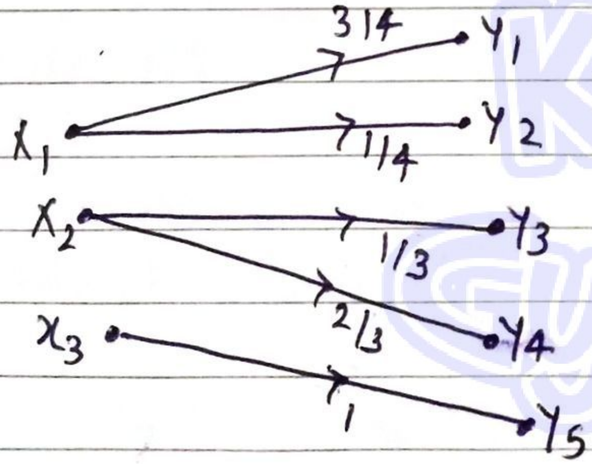
Lossless channel :—

A channel described by a channel matrix with only non zero element in each column is called lossless channel. An example of lossless channel.

$$[P(Y|X)] = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



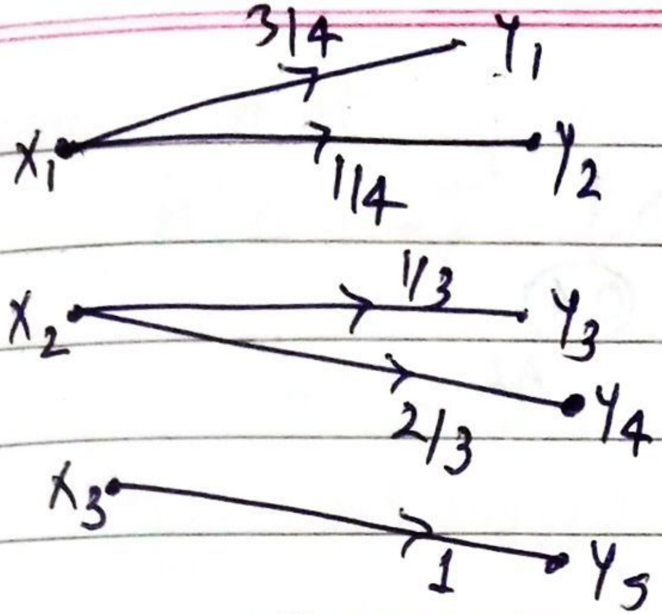
It can be shown that in the lossless channel, no source information is lost in transmission.

Prove that

$$H(X|Y) = 0 \quad \text{—— (1)}$$

When we observe the output y_j in a lossless channel shows in figure, it is clear which x_i was transmitted This means that
Further, we know that

$$P(x_i|y_j) = 0 \text{ or } 1 \quad \text{—— (2)}$$



$$H(X|Y) = -\sum_{j=1}^{n} \sum_{i=1}^{m} p(x_i, y_j) \log_2 p(x_i|y_j)$$

or

$$H(X|Y) = -\sum_{j=1}^{n} P(y_j) \sum_{i=1}^{m} P(x_i|y_j) \log_2 p(x_i|y_j)$$

Note all terms in inner summation are zero because they are in form of

$$1 \times \log_2 1 \text{ or } 0 \times \log_2 0.$$

Hence, we conclude that for a lossless channel $\boxed{H(X|Y) = 0}$

Different type of Channel:—

Lossless channel:— A channel described by a channel matrix with only one non zero element in each column is called lossless channel.

eg-

$$P(Y|X) = \begin{bmatrix} 3/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In the lossless channel, no source information is lost in transmission.

(ii) <u>Deterministic Channel:</u> — It may be noted that since each row has only one non zero element, therefore this element must be unity. Thus, when a given source symbol is sent in deterministic channel, it is clear which output symbol will be received.

$$P[y|x] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) <u>Noiseless channel:</u> — A channel is called noiseless if it is both lossless and deterministic. The input & output alphabet are of same size, that is $m = n$ for noiseless channel

eg -
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(iv) Binary Symmetric Channel [BSC]: — The BSC is defined by channel diagram shown in figure.

$$P(Y|X) = \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$$

eg - ($x_1 = 0$, $x_2 = 1$) input & output ($y_1 = 0$, $y_2 = 1$)

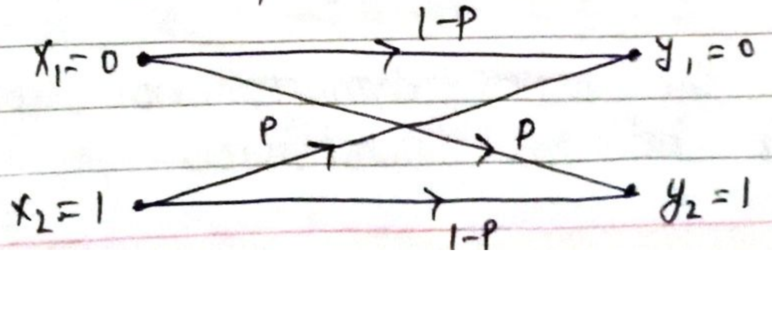


fig:- BSC

ITC :—

$$\boxed{I_k = \log_2\left(\frac{1}{p_k}\right)}$$ Amount of info.

$$H(x) = E[I(x_i)] = \sum p(x_i) I(x_i)$$

$$\sum p(x_i) \log_2 p\left(\frac{1}{x_i}\right)$$

$$\boxed{H(x_i) = -\sum p(x) \log_2 p(x_i)}$$ Entropy

Chain Rule of Entropy :—

$$H(X_1, X_2) = H(X_1) + H(X_2 | X_1)$$

$$H(X_1, X_2, X_3) = H(X_1) + \underbrace{H(X_2, X_3 | X_1)}$$

$$H(X_1) + H(X_2|X_1) + H(X_3|X_2, X_1)$$

Rate of info.

$$\boxed{R = rH}$$

$$\boxed{r = \frac{1}{t_b}}$$

$$= \sum_{i=1}^{n} H(X_i | X_{i-1}, \ldots X_1)$$

Conditional Entropy :—

$p(x,y) = p(y|x) p(x_i)$

$$\boxed{H(Y|X) = -\sum_{x \in X,\, y \in Y} p(x,y) \log \frac{p(x,y)}{p(x)}}$$

$$\boxed{H(Y|X) = -\sum_{x\in X, y\in Y} p(x,y) \log \frac{p(x,y)}{p(x)}}$$

$$H(Y|X) = \sum_{x\in X} p(x)\, H(Y|X=x)$$

$$= -\sum_{x\in X} p(x) \sum_{y\in Y} p(y|x) \log p(y|x)$$

$$= -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(y|x)$$

$$= -\sum_{x\in X\, y\in Y} p(x,y) \log p(y|x)$$

$$= -\sum_{x\in X\, y\in Y} p(x,y) \log \frac{p(x,y)}{p(x)}$$

$$= \sum_{x\in X, y\in Y} p(x,y) \log \frac{p(x)}{p(x,y)}$$

Properties of mutual information:-

(i) Symmetric (ii) $I(X;Y) \geq 0$

(iii) $\boxed{I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)}$

$H(X|Z)$ $H(X|Y_Z)$ $\boxed{H(A|B) = H(A,B) - H(B)}$

joint entropy $\boxed{I(X;Y) = H(X) + H(Y) - H(X,Y)}$

Q. A transmitter has an $\{a_1, a_2, a_3, a_4, a_5\}$ & receiver has $\{b_1, b_2, b_3, b_4\}$. The joint probabilites of system are shown as. Find Mutual information

$$P(A,B) = \begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ \hline a_1 & 0.25 & 0 & 0 & 0 \\ a_2 & 0.10 & 0.30 & 0 & 0 \\ a_3 & 0 & 0.05 & 0.10 & 0 \\ a_4 & 0 & 0 & 0.05 & 0.10 \\ a_5 & 0 & 0 & 0.05 & 0 \end{array}$$

$I(A,B) = H(A) + H(B) - H(A,B)$

$H(A) = \sum_{i=1}^{n} P(a_i) \log_2\left(\frac{1}{Pa_i}\right)$

$P(a_1) = 0.25$

$P(a_2) = 0.40$

$P(a_3) = 0.15$

$P(a_4) = 0.15$

$P(a_5) = 0.05$

$H(A) = 0.25 \log_2\left(\frac{1}{0.25}\right) + 0.40 \log_2\left(\frac{1}{0.40}\right) + 2 \times 0.15 \log_2\left(\frac{1}{0.15}\right) + 0.05 \log_2\left(\frac{1}{0.05}\right)$

$\boxed{H(A) = 2.066 \text{ b/sy}}$

$H(B) \Rightarrow$

$P(b_1) = 0.35$

$P(b_2) = 0.35$

$P(b_3) = 0.20$

$P(b_4) = 0.10$

$H(B) = 0.35 \times 2 \log_2\left(\frac{1}{0.35}\right) + 0.20 \log_2\left(\frac{1}{0.20}\right) + 0.10 \log_2\left(\frac{1}{0.10}\right)$

$\boxed{H(B) = 1.857 \text{ b/sy}}$

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$$H(A,B) = 0.25\log_2\left(\frac{1}{0.25}\right) + 3\times 0.10\log_2\left(\frac{1}{0.10}\right) + 0.30\times\log_2\left(\frac{1}{0.30}\right)$$

$$+ \; 3\times 0.05\log_2\left(\frac{1}{0.05}\right) \Rightarrow \boxed{H(A,B) = 2.666 \text{ b/s}}$$

$$I(A,B) = H(A) + H(B) - H(A,B)$$

$$I(A,B) = 2.066 + 1.857 - 2.666$$

$$\boxed{I(A,B) = 1.257 \text{ b/s}} \text{ Ans}$$

Huffman Coding :—

It uses a specific method for choosing representation for each symbol, resulting in a prefix code. It is such a widespread method for creating prefix codes that term "Huffman code" is widely used as a synonym for "prefix code" even when such a code is not produced by Huffman's algorithm.

Eg-

Symbol	Probability	code
a1	0.2	000
a2	0.4	001
a3	0.2	010
a4	0.1	011
a5	0.1	100

length of info. :-

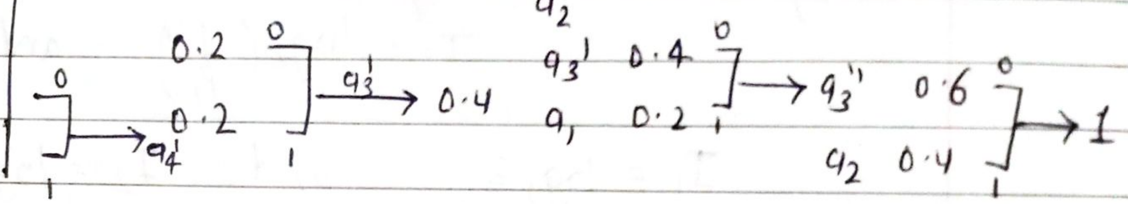
total no. of bits $= 3$

$3 \times 0.2 + 3 \times 0.4 + 3 \times 0.2 + 3 \times 0.1 + 3 \times 0.1$

$= 3$ bit /symbol

I^{st} method (Arrange the no. as per decreasing probability)

a2	0.4
a1	0.2
a3	0.2
a4	0.1
a5	0.1



Arrange value as per decreasing order

a1	0.2	01
a2	0.4	1
a3	0.2	000
a4	0.1	0010
a5	0.1	0011

Length of $inf^n =$

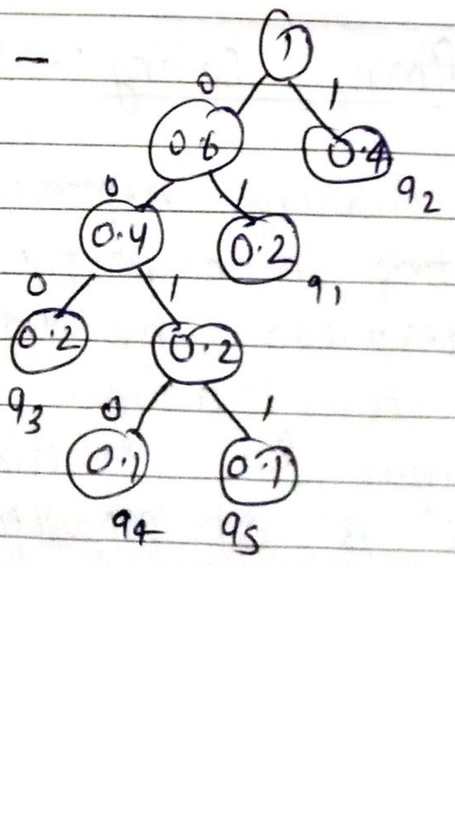
$2 \times 0.2 + 1 \times 0.4 + 3 \times 0.2 +$

$4 \times 0.1 + 4 \times 0.1$

$= \boxed{2.2 \text{ bits}}$

II^{nd} method (Tree) :-

S	P	Code
a_1	0.2	01
a_2	0.4	1
a_3	0.2	000
a_4	0.1	0010
a_5	0.1	0011



Q.

a	$P(a_i)$	Code
a1	0.81	0
a2	0.09	10
a3	0.09	110
a4	0.01	111

$\eta = ?$ $\gamma = ?$

SOURCE CODING

$\eta = \frac{H(X)}{L}$

$L = \sum_{i=1}^{n} P(x_i) n_i$

$L = (0.81)(1) + (0.09)(2) + (0.09)(3) + (0.01)(3)$

$\boxed{L = 1.29 \ b/sy}$

$\eta = \frac{0.938}{1.29} = 0.727 = 72.7\%$

$\boxed{\eta = 72.7\%}$

$H(x) = -\sum P(x_i) \log_2 [P(x_i)]$

$\gamma = 1 - \eta$

$\gamma = 1 - 0.727$

$\boxed{\gamma = 27.3\%}$

$H(x) = -(0.81 \log_2 0.81 + 0.09 \log_2 0.09 + 0.09 \log_2 0.09 + 0.01 \log_2 0.01)$

$\boxed{H(x) = 0.938 \ b/sym}$

DMC :-

Channel matrix $\Rightarrow$

$$P(Y|X) = \begin{bmatrix} P(y_1|x_1) & P(y_2|x_1) & \cdots & P(y_n|x_1) \\ \vdots & \vdots & & \\ P(y_1|x_m) & P(y_2|x_m) & \cdots & P(y_n|x_m) \end{bmatrix}$$

$\boxed{\sum_{j=1}^{n} P(y_j|x_i) = 1}$

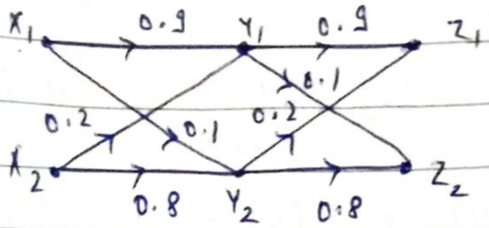
$P(X) = [P(x_1) \ \ P(x_2)]$

$P(Y) = [P(y_1) \ P(y_2) \ \cdots P(y_n)]$

$\boxed{P(Y) = [P(X)] [P(Y|X)]}$

$\boxed{P(X,Y) = [P(X)]_d [P(Y|X)]}$

Q. Two binary channel are connected in cascaded



(i) Find overall channel matrix of resultant channel & draw resultant equalent channel diagram.

(ii) find $P(z_1)$ & $P(z_2)$ when $P(x_1) = P(x_2) = 0.5$

Ans

$$P(Y|X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \qquad P(Z|Y) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

$P(Z|X) = ?$

$$P(Y) = P(X)\,P(Y|X) \quad = P(Y)\,P(Z|Y)$$

$$P(Z) = P(X)\,\underbrace{P(Y|X)\,P(Z|Y)} = P(X).P(Z|X)$$

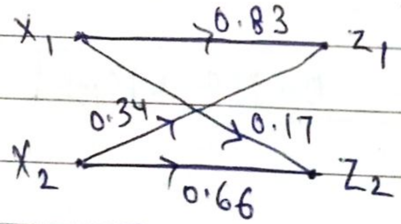
$$P(Z|X) = P(Y|X)\,P(Z|Y)$$

$$P(Z|X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

$$P(Z|X) = \begin{bmatrix} 0.83 & 0.17 \\ 0.34 & 0.66 \end{bmatrix}$$

$P(Z) = P(X)\,P(Z|X)$

$P(Z) = [0.5 \;\; 0.5] \begin{bmatrix} 0.83 & 0.17 \\ 0.34 & 0.66 \end{bmatrix}$



$P(Z) = [0.585 \;\; 0.415]$
$\qquad\quad P(z_1) \quad P(z_2)$

$\boxed{P(z_1) = 0.585}$

$\boxed{P(z_2) = 0.415}$

Lampel Ziv coding ⇒

String :-

1010 11011 01010 1010

⇓

1, 0, 10, 11, 01, 101, 010, 1010

Directory	001	010	011	100	101	110	111	1000
Content	1	0	10	11	01	101	010	1010
Code	0001	0000	0010	0011	0101	0111	1010	1100

$\Rightarrow \boxed{000100000001000110101011110101100}$

Channel Capacity $\Rightarrow$ Q. SNR = 30dB, B = 3100 Hz

$10^{(SNR_{dB}/10)}$ $\qquad (SNR)\,dB = 10\log(S/N)$

$\Rightarrow 10^{30/10} = 10^3 = 1000 \qquad 30\,dB = 10\log_{10}(S/N) \Rightarrow$

$S/N = 1000$

$C = B\log_2\left(1 + \frac{S}{N}\right) = 3100\log_2(1 + 1000)$

$= 3100\log_2(1001) = 30{,}894 = 30.89\,kbps$

Q.5 Find Efficiency & Redundance

a1	$P(a_i)$	code
$a1 = x1x1$	0.81	0
$a2 = x2x2$	0.09	10
$a3 = x2x1$	0.09	110
$a4 = x2x2$	0.01	111

Ans- We have

$$L = \sum_{i=1}^{4} p(a_i) n_i = 0.81(1) + 0.09(2) + 0.09(3) + 0.01(3)$$

$$= 1.29 \text{ b/symbol}$$

The entropy of second order extension of X, $H(X^2)$ is given by

(Note $H(X^2) = 2H(X)$)

$$H(X^2) = -\sum_{i=1}^{4} p(a_i) \log_2 p(a_i)$$

$$= -0.81 \log_2 0.81 - 0.09 \log_2 0.09 - 0.09 \log_2 0.09 - 0.01 \log_2 0.01$$

$$H(X^2) = 0.938 \text{ b/symbol}$$

Therefore code efficiency η is

$$\eta = \frac{H(X^2)}{L} = \frac{0.938}{1.29} = 0.727 = 72.7\%$$

Also, the code redundancy r will be

$$r = 1 - \eta = 1 - 0.727 = 0.273$$

$$r = 27.3\%$$

$\boxed{\eta = 72.7\%}$ $\boxed{r = 27.3\%}$

Ans - First arrange the no. in decreasing order

X_i	$P(X_i)$					
X_6	0.3 (00)	0.3 (00)	0.3 (00)	0.3 (00)	0.4 (1)	0.6 (0)
X_3	0.2 (10)	0.2 (10)	0.2 (10)	0.3 (01)	0.3 (00)	0.4 (1)
X_2	0.15 (010)	0.15 (010)	0.2 (11)	0.2 (10)	0.3 (01)	
X_5	0.15 (011)	0.15 (011)	0.15 (010)	0.2 (11)		
X_7	0.1 (110)	0.1 (110)	0.15 (011)			
X_1	0.05 (1110)	0.1 (111)				
X_4	0.05 (1111)					

Xi	Code	Codeword length
X_1	1110	4
X_2	010	3
X_3	10	2
X_4	1111	4
X_5	011	3
X_6	00	2
X_7	110	3

Average of CL

$$= \frac{4+3+2+4+3+2+3}{7}$$

$= 3$ Ans

Length of information $= 4\times0.05 + 3\times0.15 + 2\times0.2 + 4\times0.05$

$+ 3\times0.15 + 2\times0.3 + 3\times0.1$

$= 2.6$ bit/symbol

$$H(x) = \sum_{i=1}^{n} P_i \log\left(\frac{P_i}{[illegible]}\right)$$

$$= 0.05 \log_2\left(\frac{0.05}{[illegible]}\right) + 0.15 \log_2\left(\frac{0.15}{[illegible]}\right) + 0.2 \log_2\left(\frac{0.2}{[illegible]}\right)$$

$$+ 0.05 \log_2\left(\frac{0.05}{[illegible]}\right) + 0.15 \log_2\left(\frac{0.15}{[illegible]}\right) + 0.3 \log_2\left(\frac{0.3}{[illegible]}\right)$$

$$+ 0.1 \log_2\left(\frac{0.1}{[illegible]}\right)$$

$$= \cancel{2\left[0.05 \log_2\left(\frac{1}{0.05}\right) + 0.15 \log_2\left(\frac{1}{0.15}\right)\right] + 0.3 \log_2\left(\frac{1}{0.3}\right)}$$

$$\cancel{+ 0.2 \log_2\left(\frac{1}{0.2}\right) + 0.1 \log_2\left(\frac{1}{0.1}\right)}$$

$$= \quad H(x) = 2.54 \text{ bit/symbol}$$

$$\eta = \frac{H(x)}{L} = \frac{2.54}{2.6} = 0.97$$

$$\boxed{\eta = 97\%}$$

$$\gamma = 1 - \eta$$

$$\gamma = 1 - 0.97$$

$$\gamma = 0.03$$

$$\boxed{\gamma = 30\%}$$

Shannon-Fano Coding:-

Q.

X_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
P_i	1/4	1/8	1/16	1/16	1/16	1/4	1/16	1/8

Ans - Arrange X_i in Decreasing Order:-

X_i	P_i				
X_1	1/4	0	0		
X_6	1/4		1		
X_2	1/8		0	0	
X_8	1/8			1	
X_3	1/16			0	0
X_4	1/16	1	1		1
X_5	1/16			1	0
X_7	1/16				1

$$\boxed{H(x) = -\sum P(x_i) \log_2 P(x_i)}$$

$$\boxed{\text{Length of information} = \sum P(x_i)\, n_i}$$

$$\boxed{\eta = \frac{H}{L}}$$

$$\boxed{\gamma = 1 - \eta}$$

Linear Block Codes

Generator Matrix :- $G = [I_k \vdots P]_{k \times n}$

Parity check matrix :- $H = [P^T | I_{n-k}]$

Parity matrix :- P

$$p_i = \text{Rem}\left[\frac{x^{n-k+i-1}}{g(x)}\right]$$

Identity matrix :- I

Codeword $C = DG$ [code vectors]

Syndrome :- $S = CH^T$

Error vector :- $e = R - C$

Received vector $= x = c \oplus e$ Then $S = xH^T$

Q.3 What is prefix code? Explain with example.

Ans – A prefix code is a variable length code in which no codeword is prefix of another one. The codes are assigned in such a way that the code assigned to one character is not prefix of code assigned to any other character.

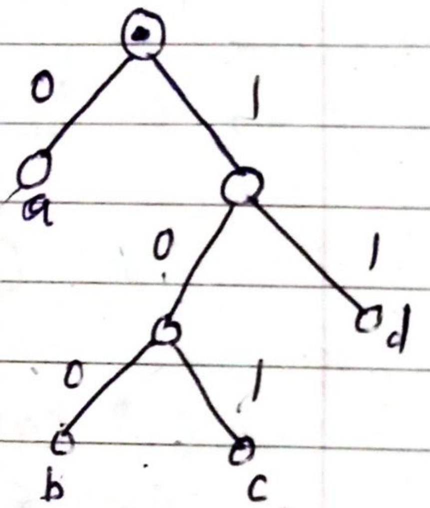
Eg – a = 0

b = 100

c = 101

d = 11

Can be viewed as a binary trie

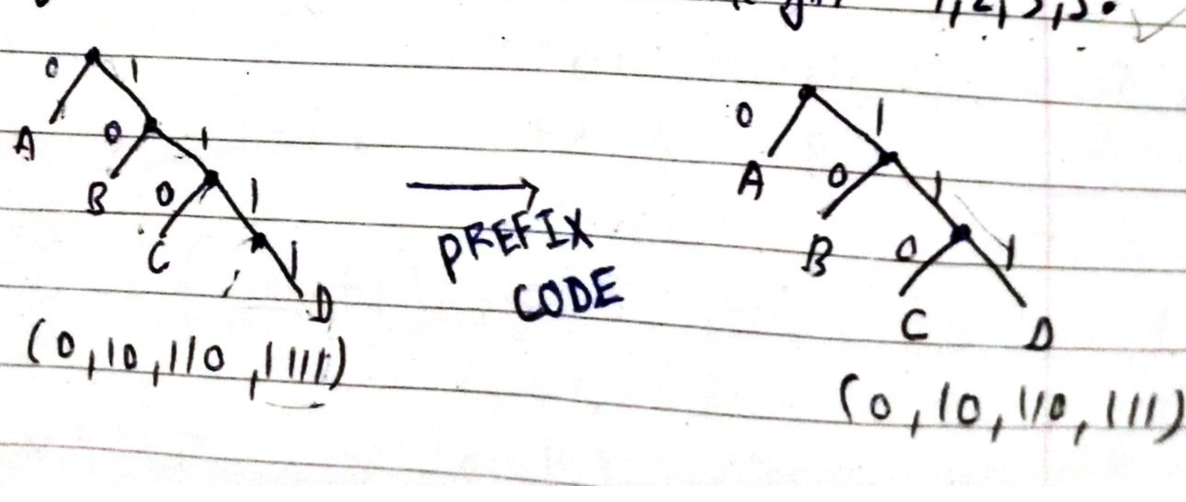


Eg – Consider prefix code

0, 10, 110, 1111.

The length of the codewords are 1, 2, 3, 4.

However length of last codeword can be reduced from 4 to 3 as (0, 10, 110, 111) is also prefix code with codeword length 1, 2, 3, 3.



What do you mean by Channel Capacity?

- Channel capacity is the tight upper bound on the rate at which information can be reliably transmitted over a communication channel.

$$W \rightarrow \boxed{\text{Encoder}} \xrightarrow{x^n} \boxed{\text{Channel}} \xrightarrow{y^n} \boxed{\text{Decoder}} \rightarrow \hat{W}$$

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

C = capacity
B = Bandwidth of channel
S = signal power
N = noiser power

when $B \to$ infinity, capacity saturates to $1.44S/N$

SNR is signal to noise power ratio.

Eg- SNR = 1000
B = 3.4KHz

$$C = B\log_2\left(1+\frac{S}{N}\right)$$

$$C = 3.4\times10^3 \log_2(1+1000)$$

$$C = 3400 * \log_2(1001)$$

$$C = (3400)(9.97) = 34\text{kbps}.$$

Q.5 The Given string = "AABABBBABAABABBBABBABBA"
Find encoding & decoding process.

Ans-

Length = 23

ASCII :- A - 65 : 01000001
B - 66 : 01000010

character	Count	Code
A	10	1
B	13	0

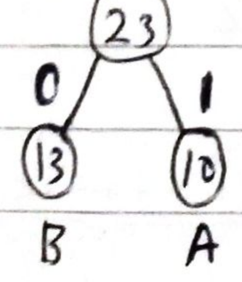
Arrange it in decreasing order

B 13 (0)
A 10 (1) → 23

Now on encoding the string
11010001011010001001001

length of information: $10 \times 1 + 1 \times 13 = 23$ bits

Now on decoding the message



Using tree method :-
"AABABBBABAABABBBABBABBA".

X	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
P	1/4	1/8	1/16	1/16	1/16	1/4	1/16	1/8

Apply Shannon-fano coding.

Arrange the X_i in decreasing order.

Symbols	P_i				
X_1	1/4	A 0		0	
X_6	1/4			1	
X_2	1/8	B 1	0	0	
X_8	1/8			1	
X_3	1/16		1	0	0
X_4	1/16				1
X_5	1/16			1	0
X_7	1/16				1

X_i	Codeword	length
X_1	00	2
X_2	100	3
X_3	1100	4
X_4	1101	4
X_5	1110	4
X_6	01	2
X_7	1111	4
X_8	101	3

Average codeword length $= \dfrac{2+3+4+4+4+2+4+3}{8}$

$= \dfrac{26}{8} = 3.25$ bit

$$H = 2\left(\frac{1}{4}\log_2 4\right) + 2\left(\frac{1}{8}\log_2 8\right) + 4\left(\frac{1}{16}\log_2 16\right)$$

$$H = \frac{4}{4}\log_2 2 + \frac{8}{8}\log_2 2 + \frac{16}{16}\log_2 2$$

$$H = 2 + 0.75 = 2.75 \text{ bits/symbol}$$

Length of information $= \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4 + \frac{1}{16}\times 4$

$$+\frac{1}{16}\times 4 + \frac{1}{4}\times 2 + \frac{1}{16}\times 4 + \frac{1}{8}\times 3$$

$$= 2.75 \text{ bits/symbol}$$

$$\eta = \frac{H}{L} = \frac{2.75}{2.75} = 1$$

$$\boxed{\eta = 100\%}$$

$$Re = 1 - \eta = 0 \quad \boxed{Re = 0\%}$$

Q.1 Given generator Matrix

$G = [1 \ 1 \ 1]$

Construct a $(x,1)$ code. How many error can this code correct. Find codeword corresponding to dataword $d=0$ & $d=1$.

Ans - Since x is a variable which can assume any value, we take a particular case of $x=3$. So required code becomes a $(3,1)$ code whose data words are (0) & (1)

$C = DG$ are

$C = (0)(111) = (000) \quad \{\because D = (0)\}$

$C = (1)(111) = (111) \quad \{\because D = (1)\}$

Since there are only two codewords & distance b/w them is 3. therefore

$d = 3 = 2t + 1$

$t = \frac{3-1}{2} \Rightarrow \boxed{t = 1}$

Code can correct all single error patterns.

Q.2 Give $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$ Find out all possible code vectors.

Ans - As we know

$G = [I_K : P]_{K \times n}$

so $k = 3$ are message bits and
$2^3 = 8$ possible combination of codeword
which is given as
(000) (001) (010) (011) (100) (101) (110) (111)

$$C = DG$$

For $D = 111$

$$C = (111)\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$C = (111000)$$

Likewise

D(Message)	Code	Vectors
000	000	000
001	001	110
010	010	101
011	011	011
100	100	011
101	101	101
110	110	110
111	111	000

Q.3 Consider (7,4) block code generated by

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Find out error vector?

Ans – The matrix P is given by

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and
$$P^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

Observing the fact that $-1 = 1$ for case of binary, we can write parity check matrix as $H = [-P^T \mid I]$

Step–① $$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Step–② $k = 4, \quad n = 7$

$2^k = 2^4 = 16$ messages $(0000) \ldots (1111)$

Step–③ $C = DG$

$$C = (1011) \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} = (1011001)$$

$1 \oplus 1 = 0$
$1 \oplus 0 = 1$

Step ④ Calculate syndrome

$$S = CH^T = (1011001) \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$S = (1011001)\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix} = [0\,0\,0]$$

Syndrome is zero, it means it is a valid code word

Step ⑤ If $R = 1001001$

$S = RH^T$

$$S = (1001001)\begin{bmatrix}1&1&1\\1&1&0\\1&0&1\\0&1&1\\1&0&0\\0&1&0\\0&0&1\end{bmatrix} = (101)$$

$S = (101)$

Step ⑥ Compare S by H^T. Now 101 is equal to third row of H^T. It means third bit from left is in error. Make it 1 if 0 and 0 if it is 1.

So transmitted word is

$C = (1011001)$

Error vector

$E = R - C$

Received Vector
$x = C \oplus e$
$\Rightarrow S = xH^T$

$$\begin{array}{rl} E = 1001001 & (R) \\ -1011001 & (C) \\ \hline 0010000 & \end{array}$$

$\boxed{E = 0010000}$

Q.4 Find linear block code encoder G if code
* generator polynomial $g(x) = 1+x+x^3$ for
a (7,4) code , $r = n-k = 3$

Ans -

$$P_1 = Re\left[\frac{x^3}{1+x+x^3}\right] = 1+x \rightarrow [1 1 0]$$

$$P_2 = Re\left[\frac{x^4}{1+x+x^3}\right] = x+x^2 \rightarrow [0 1 1]$$

$$P_3 = Re\left[\frac{x^5}{1+x+x^3}\right] = 1+x+x^2 \rightarrow [1 1 1]$$

$$P_4 = Re\left[\frac{x^6}{1+x+x^3}\right] = 1+x^2 \rightarrow [1 0 1]$$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$G = [\, I_k : P \,]$$

$$G = \left[\begin{array}{cccc:ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$H = [\, P^T ; I_{n-k} \,] = \left[\begin{array}{cccc:ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \left[\begin{array}{cccc:ccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right]^T$$

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For convenience the code vector is expressed as

$C = [\, m \mid c_p \,]$ where $c_p = mP$ is an $(n-k)$ bit parity check vector

If $m = 1010$

$$m = 1 + x^2$$

$$g(x) = 1 + x + x^3$$

$$C = m \cdot g(x) = (1+x^2)(1+x+x^3)$$

$$C = 1 + x + x^3 + x^2 + x^3 + x^5$$

$$C = 1 + x + x^2 + x^5$$

$$C = (1110010)$$

So

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Q-5 G (6,3) block code is shown below. Obtain all code vector of this code?

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Sol- As we know

$$G = [I_k : P]_{k \times n}$$

$$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Here $k=3$ & $n=6$

$2^k = 2^3 = 8$ message

S.N.	Bit of message vector		
	m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

$$[C_1\ C_2\ C_3] = [m_1\ m_2\ m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$C_1 = (0 \times m_1) + m_2 + m_3 = m_2 \oplus m_3$ —(1)

$C_2 = m_1 \oplus m_3$ —(2) $\quad C_3 = m_1 \oplus m_2$ —(3)

$(m_1 m_2 m_3) = (0,0,0)$

$C_1 = 0 \oplus 0 = 0$

$C_2 = 0 \oplus 0 = 0$

$C_3 = 0 \oplus 0 = 0$

$(C_1 C_2 C_3) = 000$

$(0\,0\,1) \Rightarrow \quad C_1 = 0 \oplus 1 = 1$

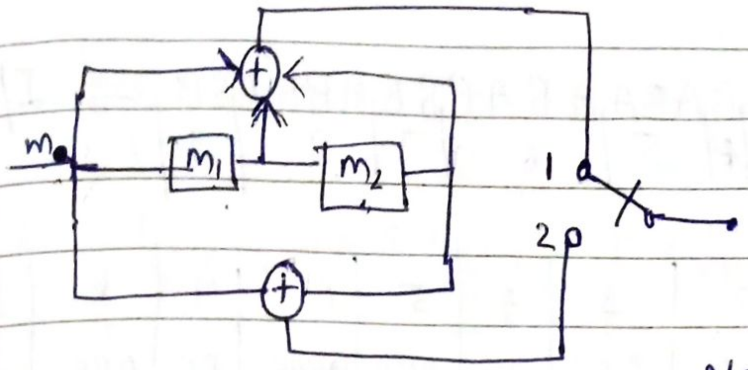
$C_2 = 0 \oplus 1 = 1 \quad , \quad C_3 = 0 \oplus 0 = 0$

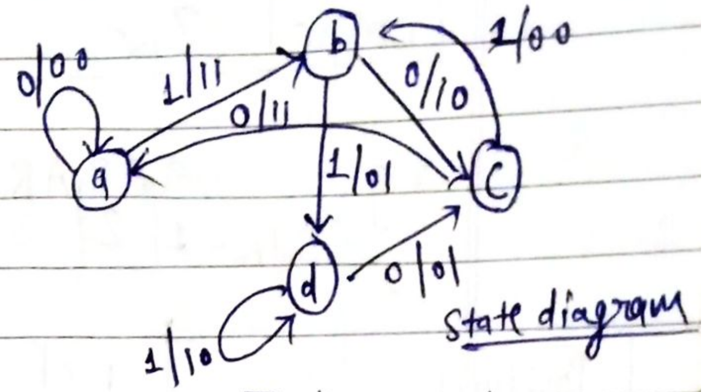
$(C_1 C_2 C_3) = 110$

Likewise

D (Message)	Code	Vectors
000	000	000
001	001	110
010	010	101
011	011	011
100	100	1011
101	101	101
110	110	110
111	111	000

Convolution Codes $\Rightarrow$





State diagram

Trellis Encoder

m_0	m_1	I/P m_2	PS	NS	O/P X_1	X_2
0	0	0	a	a	0	0
0	0	1	a	b	1	1
0	1	0	b	c	1	0
0	1	1	b	d	0	1
1	0	0	c	a	1	1
1	0	1	c	b	0	0
1	1	0	d	c	0	1
1	1	1	d	d	1	0

$X_1 = m_0 \oplus m_1 \oplus m_2$

$X_2 = m_0 \oplus m_2$

a = 00

b = 01

c = 10

d = 11

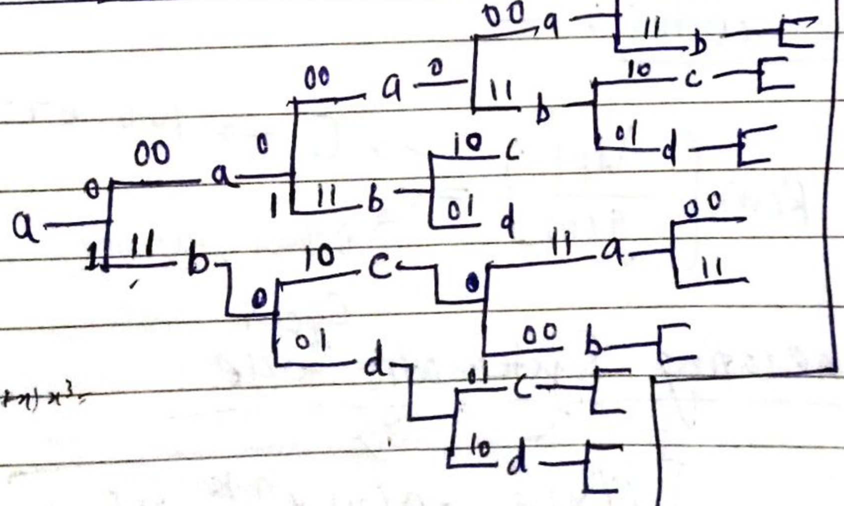
$0 \oplus 0 = 0$

$0 \oplus 1 = 1$

$1 \oplus 0 = 1$

$1 \oplus 1 = 0$

Code Tree [Trellis Tree] $\Rightarrow$



$(x^3+x)x^3=$

$x^3+x+1 \overline{) x^6+x^4} (x^3+1$

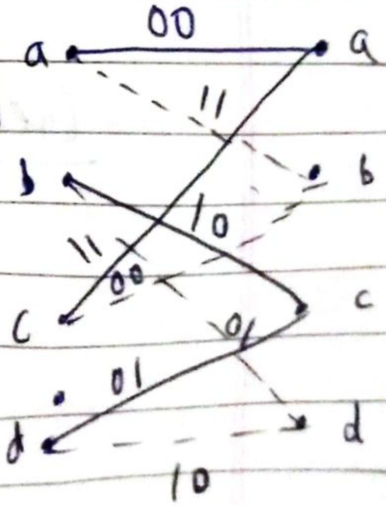
$\cancel{x^6+x^4+x^3}$

x^3

x^3+x+1

$x+1 \quad 011$

Trellis Diagram $\Rightarrow$



1, 2, 3, 5, 7, 11, 13 ---

Galois Field ⇒

Galois showed that for a field to be finite, the no. of elements should be p^n, where p is a prime & n is a positive integer

GF(2)

{0, 1}

$0^0 =$

$\cancel{0^1 = 0}$

$1^0 = 1$

GF(3)

{0, 1, 2}

$2^0 = 1$

$2^1 = 2$

$2^2 = 4 \bmod 3 = 1$

GF(5) {0, 1, 2, 3, 4}

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8 \bmod 5 = 3$

$2^4 = 16 \bmod 5 = 1$

$3^0 = 1$

$3^1 = 3$

$3^2 = 9 \bmod 5 = 4$

$3^3 = 27 \bmod 5 = 2$

$3^4 = 81 \bmod 5 = 1$

GF(2) ⇒

Addition

+	0	1
0	0	1
1	1	0

Multiplication

*	0	1
0	0	0
1	0	1

G

Additive Inverse

a	0	1
-a	1	0

Multiplicative Inverse

a	0	1
a^{-1}	–	1

GF(5) ⇒

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

*	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Additive Inverse

a	0	1	2	3	4
-a		4	3	2	0

Multiplicative Inverse

a	0	1	2	3	4
a^{-1}	-	1	3	2	4

GF(5)

Additive Inverse

a	0	1	2	3	4
-a	0	4	3	2	1

$\frac{1 \times 1}{5} = 1$ $\quad \frac{1}{5} = 1$

$2 \times 3 = \frac{6}{5} = 1$

$4 \times 4 = \frac{16}{5} \mod 5 = 1$

$\frac{5}{5} =$

$1 + 4 = 5 - 5 = 0$ ✓

$\Rightarrow G(2^n)$ $\quad$ n is degree

Degree	Irreducible Polynomial
1	$x, (x+1)$
2	$1 + x + x^2$
3	$(x^3 + x^2 + 1), (x^3 + x + 1)$
4	$(x^4 + x^3 + x^2 + x + 1), (x^4 + x^3 + 1), (x^4 + x + 1)$
5	$(x^5 + x^2 + 1), (x^5 + x^3 + x^2 + x + 1), \ldots$

Q. $GF(2^2)$ $\quad \{00, 01, 10, 11\} \Rightarrow \{0, 1, x, x+1\}$

$2^n \Rightarrow 2^2 \Rightarrow \boxed{n = 2}$ $\quad IP = x^2 + x + 1$

+	00	01	10	11
00	00	01	10	11
01	01	00	11	10
10	10	11	00	01
11	11	10	01	00

*	00	01	10	11
00	00	00	00	00
01	00	01	10	11
10	00	10	11	01
11	00	11	01	10

$x \times x + 1$

$= x^2 + x$

$x^2 + x + 1$

$x^2 + x + 1 \overline{) \; x^2 + x \;} 1$

$\quad \cancel{x^2 + x + 1}$

$\quad 1$

Polynomials :-

$$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x^1 + a_0x^0$$

Eg- n bit word = 10011001

	1	0	0	1	1	0	0	1
	↓	↓	↓	↓	↓	↓	↓	↓
Polynomial	$1x^7$	$0x^6$	$0x^5$	$1x^4$	$1x^3$	$0x^2$	$0x$	1.1

→ $\boxed{x^7 + x^4 + x^3 + 1}$

→ Addition & subtraction operations are same on polynomial

Eg- $-x$ equal to x as:-

$-x - x + x$

$-2x + x$

$0 + x = x$ ✓

Q. $(x^5 + x^2 + x) \oplus (x^3 + x^2 + 1)$ $GF(2^8)$ x^7

$$\begin{array}{rl} \oplus & 0x^7 + 0x^6 + 1x^5 + 0x^4 + 0x^3 + 1x^2 + 1x + 0.1 \\ & 0x^7 + 0x^6 + 0x^5 + 0x^4 + 1x^3 + 1x^2 + 0x + 1.1 \\ \hline & 0 \quad 0 \quad 1x^5 + 0 + 1x^3 + 0 + x + 1 \end{array}$$

→ $\boxed{x^5 + x^3 + x + 1}$

Q. $(x^5 + x^2 + x) \otimes (x^7 + x^4 + x^3 + x^2 + x)$ $GF(2^8)$

IP: $(x^8 + x^4 + x^3 + x + 1)$

A-

$x^5(x^7 + x^4 + x^3 + x^2 + x) + x^2(x^7 + x^4 + x^3 + x^2 + x) + x(x^7 + x^4 + x^3 + x^2 + x)$

Er. Sahil ka Gyan

Errors
- Random
  - Gaussian Noise
    - Thermal
    - Shot Noise
- Burst

Classification of codes
- Error Detecting Codes
- Error correcting code
  - Block Codes
  - Convolution Codes

⟹ for error-free transmission

codes
- Block Codes
  - for Error Correction
  - for Error Detection
- Burst & Random Error Correcting Codes
- Interleaving
  - Block Interleaving
  - Convolutional Interleaving

⟹ An Another classification of codes

- Instantaneous codes
- Optimal Codes

(c) Viterbi Algorithm performs step by step as follow:-

(1) Initilization:- Set the metric of left most state of the trellis at 0.

(2) Computation step j+1:
We suppose that at previous step we have identified all survivor path & stored each state's survior path. For each state at level j+1, we compute metric of incoming paths as addition of metric of incoming branch & metric of survior path. We then choose path with smallest metric for each state.

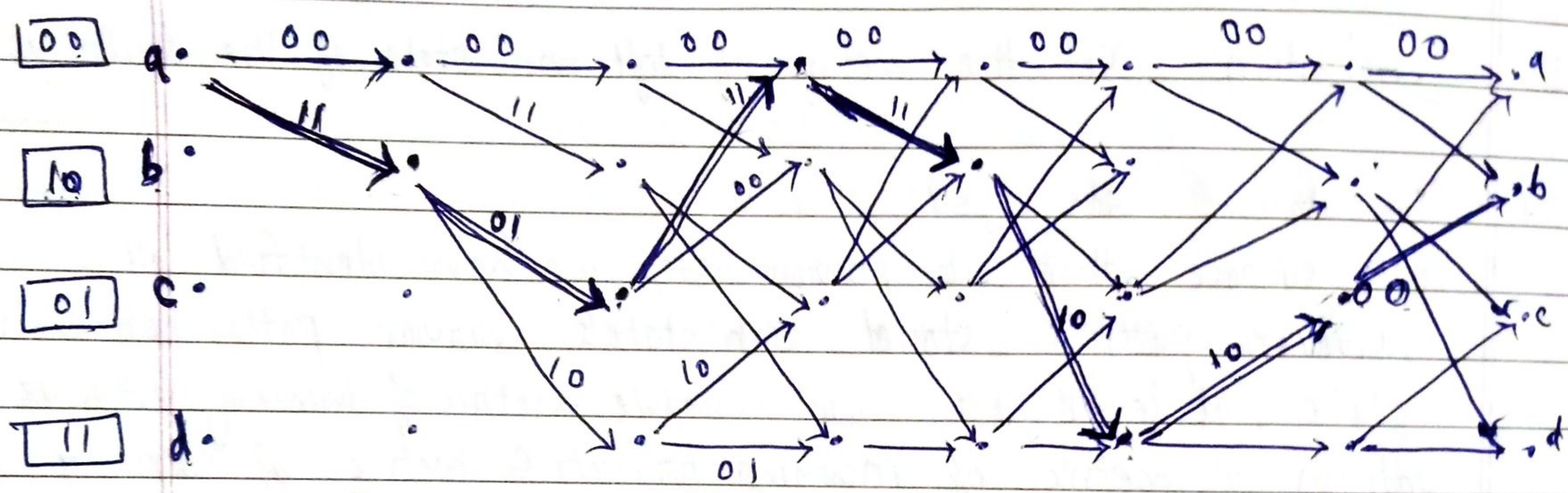
(3) Final Step:-
We continue computation until algo. reaches termination node, at which time it makes a decision on max.-likelihood path. Then decoded sequence is sequence of bits corresponding to this optimum path's branches.

m_0	m_1	m_2	x_1	x_2	PS	NS
0	0	0	0	0	a	a
0	0	1	1	1	a	b
0	1	0	1	0	b	c
0	1	1	0	1	b	d
1	0	0	1	1	c	a
1	0	1	0	0	c	b
1	1	0	0	1	d	c
1	1	1	1	0	d	d

a = 00
b = 01
c = 10
d = 11

Trellis diagram :-

Input = $\boxed{1}\boxed{0}\boxed{0}\boxed{1}\boxed{1}\boxed{0}\boxed{1}$



Output = [11 01 11 11 10 10 00] Ans

(9)

Calculation of CRC ⇒

(e) At Sender Side:—

A String of n 0's is appended to data unit to be transmitted.

Here, n is one less than no. of bit's in CRC generator.

Binary division is performed of resultant string with CRC generator.

After division, remainder so obtained is called as CRC.

~~After division remainder~~ s

It may be noted that CRC also consists of n-bits.

Appending CRC to data unit a :—

At sender side

CRC is obtained after binary division.

Generator polynomial $G(x) = x^4 + x + 1$ is encoded as 10011.

Generator polynomial consists of 5 bits.

So, a string of 4 zeroes is appended to bit stream to be transmitted.

The resulting bit stream is 11010110110000.

Binary Division is performed as -

```
10011 ) 11010110110000 ( 1100001010
        10011
         10011
         10011
          00001
          00000
           00010
           00000
            00101
            00000
             01011
             00000
              010110
               10011
               01010
               00000
                10100
                10011
                 01110
                 00000
                  1110  <- Remainder
```

[Cyclic Redundancy Check]

CRC

$\boxed{CRC = 1110}$

The code word transmitted to receiver = $\boxed{11010110111110}$

(a) $g(x) = 1+x^2+x^3$

The given generator polynomial can be written as

$$g(x) = 1+0\cdot x + x^2 + x^3$$

$g_1 = 0$

$g_2 = 1$

Comparing with equation $x^3 + g_2 x^2 + g_1 x + 1$

Encoder for generator polynomial $g(x) = 1+x^2+x^3$ Given by figure:—